The Mathematics Student ISSN: 0025-5742
Vol. 88, Nos. 1-2, January-June (2019), 137–145

ON $N(\kappa)$-PARA CONTACT 3-MANIFOLDS WITH RICCI SOLITONS

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(Received: 09 - 07 - 2018; Revised: 30 - 10 - 2018)

ABSTRACT. The present paper contains the study of $N(\kappa)$-para contact 3-manifolds with Ricci solitons. It is established that such solitons are steady. In addition, if a soliton is gradient Ricci soliton, then the manifold is flat. An example is provided to support the obtained results.

1. INTRODUCTION

It is well known that the Poincare Conjecture, posed by Henry Poincare in 1904, was a long standing unsolved problem of geometry and topology. The century long problem was solved by G. Perelman in 2002[13]. In order to solve the problem Perelman used the techniques of Ricci flow developed by R. S. Hamilton [8]. Around the same time Friedan [5] introduced the theory of Ricci flow in physics with a different motivation. In geometric topology Hamilton's Ricci flow is a heat type parabolic partial differential equation of the following form:

$$\frac{\partial g_{ij}}{\partial t} = -2S_{ij}, \qquad g_{ij}(0) = g_0. \tag{1.1}$$

A Ricci soliton is a constant solution of the Ricci flow equation upto diffeomorphism and scaling. On a Riemannian manifold M with metric g, a Ricci soliton is expressed by

$$(\text{Ł}_V g)(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0. \tag{1.2}$$

Here Ł_V denotes the Lie derivative operator along a complete vector field V which is known as a potential vector field. It is assumed that V is complete. Here λ is a constant, known as soliton constant and S is the Ricci curvature of the manifold M. X, Y are arbitrary vector fields of M. The Ricci soliton is said to be shrinking, steady or expanding according as λ is negative, zero

2010 Mathematics Subject Classification: 53C15, 53D25.
Key words and phrases: $N(\kappa)$-para contact metric manifolds, Ricci soliton, gradient Ricci soliton.

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